IV. *An Account of a Method of dividing Astronomical and other Instruments, by ocular Inspection; in which the usual Tools for graduating are not employed; the whole Operation being so contrived, that no Error can occur but what is chargeable to Vision, when assisted by the best optical Means of viewing and measuring minute Quantities. By Mr.* Edward Troughton. *Communicated by the Astronomer Royal.*

Read February 2, 1809.

It would ill become me, in addressing myself to the Members of this Society upon a subject which they are so well enabled to appreciate, to arrogate to myself more than may be assigned as my due, for whatever of success may have been the result of my long continued endeavours, exerted in prosecuting towards perfection *the dividing of Instruments immediately subservient to the purposes of Astronomy.* A man very naturally will set a value upon a thing on which so much of his life has been expended; and I shall readily, therefore, be pardoned for saying, that considering some attainments which I have made on this subject as too valuable to be lost, and being encouraged also by the degree of attention which the Royal Society has ever paid to practical subjects, I feel myself ambitious of presenting them to the public through what I deem the most respectable channel in the world.

It was as early as the year 1775, being then apprentice to my brother, the late Mr. John Troughton, that the art of

dividing had become interesting to me; the study of astronomy was also new and fascinating; and I then formed the resolution to aim at the nicer parts of my profession.

At the time alluded to, my brother, in the art of dividing, was justly considered the rival of RAMSDEN; but he was then almost unknown beyond the narrow circle of the mathematical and optical instrument makers; for whom he was chiefly occupied in the division, by hand, of small astronomical quadrants, and HADLEY's sextants of large radius. Notwithstanding my own employment at that time was of a much inferior nature, yet I closely inspected his work, and tried at leisure hours on waste materials to imitate it. With as steady a hand, and as good an eye, as young men generally have, I was much disappointed at finding, that after having made two points, neat and small to my liking, I could not bisect the distance between them, without enlarging, displacing, or deforming them with the points of the compasses. This circumstance gave me an early dislike to the tools then in use; and occasioned me the more uneasiness, as I foresaw that it was an evil which no practice, care, nor habit could entirely cure: Beam-compasses, spring-dividers, and a scale of equal parts, in short, appeared to me little better than so many sources of mischief.

I had already acquired a good share of dexterity, as a general workman: Of the different branches of our art, that of *turning* alone seemed to me to border on perfection. This juvenile conceit, fallacious as I afterwards found it, furnished the first train of thoughts, which led to the method about to be described; for it occurred to me, that if I could by any means apply the principle of turning to the art of dividing

instruments, the tools liable to objection might be dispensed with. The means of doing this was first suggested, by seeing the action of the perambulator, or measuring wheel; the surface of the earth presenting itself as the edge of the instrument to be divided, and the wheel of the perambulator as a narrow roller acting on that edge; and hence arose an idea that some easy contrivance might be devised, for marking off the revolutions and parts of the roller upon the instrument. Since the year above-mentioned, several persons have proposed to me, as new, dividing by the roller, and I have been told, that it also occurred long ago to HOOK, SISSON, and others; but, as HATTON on Watch-making, says, "I do not "consider the man an inventor, who merely thinks of a thing; "to be an inventor, in my opinion, he must act successfully "upon the thought, so as to make it useful." I had no occasion, however, to have made an apology for acting upon a thought, which, unknown to me, had been previously conreived by others; for it will be seen in the sequel, how little the roller has to do in the result, and with what extreme caution it is found necessary to employ it.

When a roller is properly proportioned to the radius of the circle to be divided, and with its edge made a small matter conical, so that one side may be too great, and the other side too little, it may be adjusted so exactly, that it may be carried several times around the circle, without the error of a single second; and it acts with so much steadiness, that it may not unaptly be considered as a wheel and pinion of indefinitely high numbers. Yet, such is the imperfection of the edges of the circle and roller, that, when worked with the greatest care, the intermediate parts, on a radius of two feet, will sometimes be

unequal to the value of half a minute or more. After having found the terminating point of a quadrant or circle so permanent, although I was not prepared to expect perfect equality throughout, yet I was much mortified to find the errors so great, at least ten times as much as I expected; which fact indicated, beyond a doubt, that if the roller is to be trusted at all, it must only be trusted through a very short arc. Had there been any thing slippery in the action, which would have been indicated by measuring the same part at different times differently, there would have been an end of it at once; but, that not being the case in any sensible degree, the roller becomes an useful auxiliary to fill up short intervals, whose limits have been corrected by more certain means.*

* There are two things in the foregoing account of the action of the roller which have a tendency to excite surprise. The first is, that the roller should, in different parts of its journey round the circle, measure the latter so differently. One would not wonder, however, if in taking the measure across a ploughed field, it should be found different to a parallel measure taken upon a gravel-walk; and, in my opinion, the cases are not very dissimilar. Porosity of the metal, in one part of the circle more than in the other, must evidently have the same effect; brass unhammered is always porous; and the part, which has felt the effect of two blows, cannot be so dense as other parts which have felt the effect of three; and, should the edge of the circle be indented by *jarring-turning*, it would produce a visible similitude to ploughed ground: Every workman must be sufficiently upon his guard against such a palpable source of error; yet, perhaps with our greatest care we may not be able to avoid it altogether. The second is, that notwithstanding the inequality above-mentioned, the roller having reached the point upon the circle from whence it set out, should perform a second, third, &c. course of revolutions, without any sensible deviation from its former track; this is not perhaps so easily accounted for. It must be mentioned, that the exterior border of the circle should be *turned rounding*, presenting to the roller a convex edge, whose radius of curvature is not greater than one tenth of an inch. Now, were the materials perfectly inelastic and impenetrable, the roller could only touch the circle in a *point*, and in passing round the circle, it could only occupy a *line* of contact. This in practice is not the case; the circle always marks the roller with a

BIRD, who enjoyed the undisputed reputation of being the most accurate divider of the age in which he lived, was the first who contrived the means how to render the usual divisions of the quadrant bisectional; which property, except his being unusually careful in avoiding the effects of unequal expansion from change of temperature, chiefly distinguished his method from others who divided by hand. This desirable object he accomplished by the use which he made of a finely divided scale of equal parts. The thing aimed at was, to obtain a point upon the arc at the highest *bisectional number of divisions* from 0, which in his eight feet quadrants was 1024, $= 85° 20'$. The extent of the beam-compasses, with which he traced the arc upon the limb of the instrument to be divided, being set off upon that arc, gave the points 0° and 60°;

broad list, and thereby shews that there is a yielding between them to a considerable amount. The breadth of this list is not less than one fiftieth of an inch; and it follows, that at least 12° of the circle's edge must be in contact at the same time; that the two surfaces yield to each other in depth, by a quantity equal to the *ver. sin.* of half that arc, or $\frac{1}{1800}$ of an inch; and that the circle has always hold of the roller by nearly 1° of the edge of the latter. Whoever has examined the surfaces of metals which have rolled against each other, must have observed that peculiar kind of indentation that always accompanies their action; and there can be no doubt that the particles of a roller, and those of the surface on which it acts, which mutually indent each other, will, upon a second course begun from the same point, indent each other deeper: This is not, however, exactly the case in question; for, whatever of fitting might have taken place between the surfaces of our roller and circle in the first revolution of the former, one should imagine would be obliterated by the fifteen turns which it must repeat over fresh ground. Experience shows, however, as every one will find who tries the experiment with good work, that on coming round to the point of commencement, the roller has the disposition to regain its former track; for, were this not the case, although the commensurate diameters were adjusted so exactly as to be without sensible error in one course, yet a less error than that which is so would become visible, when repeated through many courses.

which, being bisected, gave 30° more to complete the total arc. A second order of bisections gave points at 15° distance from each other; but that which denoted 75° was most useful. Now, from the known length of the radius, as measured upon the scale, the length of the chord of 10° 20′ was computed, taken off from the scale, and protracted from 75° forwards; and the chord of 4° 40′, being ascertained in the same manner, was set off from 90° backwards, meeting the chord of 10° 20′ in the continually bisectional arc of 85° 20′. This point being found, the work was carried on by bisections, and the chords, as they became small enough, were set off beyond this point to supply the remainder of the quadrantal arc. My brother, whom I mentioned before, from mere want of a scale of equal parts upon which he could rely, contrived the means of dividing bisectionally without one. His method I will briefly state as follows, in the manner which it would apply to dividing a mural quadrant. The arcs of 60° and 30° give the total arc as before; and let the last arc of 30° be bisected, also the last arc of 15°, and again the last arc of 7° 30′: The two marks next 90° will now be 82° 30′ and 86° 15′, consequently the point sought lies between them. Bisections will serve us no longer; but if we divide this space equally into three parts, the most forward of the two intermediate marks will give us 85°, and if we divide the portion of the arc between this mark and 86° 15′ also into three, the most backward of the two marks will denote 85° 30′. Lastly, if we divide any one of these last spaces into five, and set off one of these fifth parts backwards from 85° 30′, we shall have the desired point at 1024 divisions upon the arc from 0°. All the rest of the divisions which have been made in this operation, which I have

called marks because they should be made as faint as possible, must be erased; for my brother would not suffer a mark to remain upon the arc to interfere with his future bisections.

Mr. SMEATON, in a paper to be more particularly noticed presently, justly remarks the want of a unity of principle in Mr. BIRD's method; for he proceeds partly on the ground of the protracted radius, and partly upon that of the computed chord; which, as SMEATON observes, may or may not agree. BIRD, without doubt, used the radius and its parts in order to secure an exact quadrant; but SMEATON, treating exactness in the total arc as of little value to astronomy, would, in order to secure the more essential property of equality of division, reject the radius altogether, and proceed entirely upon the simple principle of the computed chord. The means pursued by my brother, to reach the point which terminates the great bisectional arc, is the only part in which it differs from BIRD's method; and, I think it is without prejudice that I give it the preference. It is obvious that it is as well calculated to procure equality of division, as the means suggested by SMEATON; at the same time that it is equal to BIRD's in securing the precise measure of the total arc. It proceeds entirely upon the principle of the protracted chord of 60° and its subdivision; and the uncertainty, which is introduced into the work by the sparing use which is made of subdivision by 3 and 5, is, in my opinion, likely to be much exceeded by the errors of a divided scale,* and those of the hand and eye, in taking off the computed chords, and applying them to the arc of the instrument to be divided.

* That BIRD's scale was not without considerable errors, will be shewn towards the end of this paper.

RAMSDEN's well known method of dividing by the engine unites so much accuracy and facility, that a better can hardly be wished for; and I may venture to say that it will never be superseded, in the division of instruments of *moderate radii.* It was well suited to the time in which it appeared; a time when the improvements made in nautical astronomy, and the growing commerce of our country, called for a number of reflecting instruments, which never could have been supplied, had it been necessary to have divided them by hand; however, as it only applies to small instruments, it hardly comes within the subject of this paper.

The method of HINDLEY, as described by SMEATON,* I will venture to predict will never be put in practice for dividing astronomical instruments, however applicable it might formerly have been for obtaining numbers for cutting clock-work, for which purpose it was originally intended. It consists of a train of violent operations with blunt tools, any one of which is sufficient to stretch the materials beyond, or press them within their natural state of rest; and, although the whole is done by contact, the nature of this contact is such as, I think, ought rather to have been contrasted with, than represented as being similar to, the nature of the contact used in SMEATON's Pyrometer, which latter is performed by the most delicate touch; and is represented, I believe justly, to be sensible to the $\frac{1}{60000}$ part of an inch. SMEATON has, however, acquitted himself well, in describing and improving the method of his friend; and the world is particularly obliged to him for the historical part of his paper, as it contains valuable information which perhaps no one else could have written.

* Phil. Trans. for 1788.

The only method of dividing large instruments now practised in London, that I know of, besides my own, has not yet, I believe, been made public. It consists in dividing by hand with beam compasses and spring dividers, in the usual way; with the addition of examining the work by microscopes, and correcting it, as it proceeds, by pressing forwards or backwards by hand, with a fine conical point, those dots which appear erroneous; and thus adjusting them to their proper places. The method admits of considerable accuracy, provided the operator has a steady hand and good eye; but his work will ever be irregular and inelegant. He must have a circular line passing through the middle of his dots, to enable him to make and keep them at an equal distance from the centre. The bisectional arcs, also, which cut them across, deform them much; and, what is worse, the dots which require correction (about two thirds perhaps of the whole) will become larger than the rest, and unequally so in proportion to the number of attempts which have been found necessary to adjust them. In the course of which operation, some of them grow insufferably too large, and it becomes necessary to reduce them to an equality with their neighbours. This is done with the burnisher, and causes a hollow in the surface, which has a very disagreeable appearance. Moreover, dots which have been burnished up are always ill-defined, and of a bad figure. Sir GEORGE SHUCKBURGH EVELYN, in his paper on the Equatorial,* denominates these "doubtful or bad points;" and, (considering the few places which he examines) they bear no inconsiderable proportion to the whole. In my opinion, it would be a great improvement of this method, to

* Phil. Trans. for 1793.

divide the whole by hand at once, and afterwards to correct the whole; for a dot forced to its place, as above, will seldom allow the compass-point to rest in the centre of its apparent area; therefore other dots made from those will scarcely ever be found in their true places. This improvement also prevents the corrected dots from being injured, or moved, by the future application of the compasses, no such application being necessary.

I will now dismiss this method of dividing, with observing, that it is tedious in the extreme; and did I not know the contrary beyond a doubt, I should have supposed it to have surpassed the utmost limit of human patience.* When I made my first essay at subdividing with the roller, I used this method, according to the improvement suggested above, of correcting a few primitive points; but even this was too slow for one who had too much to do. Perhaps, however, had my instruments been divided for me by an assistant, I might not have grudged to have paid him for the labour of going through the whole work by the method of adjustment; nor have felt the necessity of contriving a better way.

I might now extend the account of my method of dividing to a great length; by relating the alterations which the

* At the time alluded to, the double microscopic micrometer was unknown to me, and I did not learn its use, for these purposes, till the year 1790, from General ROY's description of the large theodolite. Previous to that time, I had used a frame which carried a single wire very near the surface to be divided; this wire was moveable by a fine micrometer screw, and was viewed by a single lens inserted in the lower end of a tube, which, for the purpose of taking off the parallax, was 4 inches long. The greatest objection to this mode of constructing the apparatus is, that the wire being necessarily exposed, is apt to gather up the dust; yet it is preferable to the one now in use, in cases where any doubt is entertained of the accuracy of the plane which is to receive the divisions.

apparatus has undergone during a long course of years,* and the various manner of its application, before I brought it to its present state of improvement; but I think I may save myself that trouble, for truly I do not see its use: I will, therefore, proceed immediately to a disclosure of the method, as practised on a late occasion, in the dividing of a four feet meridian circle, now the property of STEPHEN GROOMBRIDGE, Esq. of Blackheath.

The surface of the circle which is to receive the divisions, as well as its inner and outer edges, but especially the latter, should be turned in the most exact and careful manner; the reason for which will be better understood, when we come to describe the mode of applying the roller: and, as no projection can be admitted beyond the limb, if the telescope, as is generally the case, be longer than the diameter, those parts which extend further must be so applied, that they may be removed during the operation of dividing. Fig. 1 and 2 represent the principal parts of the apparatus; Fig. 1 shewing the plan, and Fig. 2 the elevation; in both of which the same letters of reference are affixed to corresponding parts, and both are drawn to a scale of half dimensions. A A is a part of the circle, the surface of which is seen in the plan, and the edge is seen in the elevation. B B B is the main plate of the apparatus, resting with its four feet *a a a a* upon the surface of the arc; these feet, being screws, may be adjusted so as to take equal shares of the weight, and then are fastened by nuts below the plate,

* The full conception of the method had occupied my mind in the year 1778; but as my brother could not be readily persuaded to relinquish a branch of the business to me in which he himself excelled, it was not until September 1785 that I produced my first specimen, by dividing an astronomical quadrant of two feet radius.

as shewn in Fig. 2. CC and DD are two similar plates, each attached to the main plate, one above and the other below, by four pillars; and in them are centred the ends of the axis of the roller E. F and G are two friction wheels, the latter firmly fastened to B, but the former is fixed in an adjustable frame, by means of which adjustment these wheels and the roller E, may be made to press; the former on the interior, and the latter on the exterior edge of the circle, with an equal and convenient force.* At the extremities of the axis of the roller, and attached to the middle of the plates C and D, are two bridges, having a screw in each; by means of which an adjustment is procured for raising or lowering the roller respecting the edge of the circle, whereby the former, having its diameter at the upper edge about .001 of an inch greater than at the lower edge (being, as before described, a little conical), it may easily be brought to the position where it will measure the proper portion of the circle.

Much experience and thought upon the subject have taught me, that the roller should be equal to one sixteenth part of the circle to be divided, or that it should revolve once in 22° 30′; and that the roller itself should be divided into sixteen parts; no matter whether with absolute truth, for accuracy is not at all essential here. Each of such divisions of the roller will correspond with an angle upon the circle of 1° 24′ 22,″5, or $\frac{1}{256}$th part of the circle. This number of principal divisions was chosen, on account of its being capable of continual bisec-

* Sufficient spring for keeping the roller in close and uniform contact with the edge of the circle, is found in the apparatus, without any particular contrivance for that purpose; the bending of the pillars of the secondary frames and of the axis of the roller, chiefly supplies this property.

tion; but they do not fall in with the ultimate divisions of the circle, which are intended to be equal to 5′ each.

The next thing to be considered is, how to make the roller measure the circle. As two microscopes are here necessary, and those which I use are very simple, I will in this place give a description of them. Fig. 6 is a section of the full size, and sufficiently explains their construction, and the position of the glasses; but the micrometer part and manner of mounting it, are better shown at H, in Fig. 1 and 2. The micrometer part consists of an oblong square frame, which is soldered into a slit, cut at right angles in the main tube; another similar piece nicely fitted into the former, and having a small motion at right angles to the axis of the microscope, has at one end a cylindrical guide pin, and at the other a micrometer screw; a spring of steel wire is also applied, as seen in the section, to prevent play, by keeping the head of the micrometer in close contact with the fixed frame. This head is divided into one hundred parts, which are numbered each way to 50; the use of which will be shewn hereafter. A fine wire is stretched across the moveable frame, for the purpose of bisecting fine dots. Two of these microscopes are necessary; also a third, which need not have the divided head, and must have in the moveable frame two wires crossing each other at an angle of about 30°; this microscope is shewn at I, Fig. 1. In the two first micrometers, a division of the head is of the value of about 0″,2, and the power and distinctness such, that when great care is taken, a much greater error than to the amount of one of these divisions cannot well be committed in setting the wire across the image of a well made dot. The double eye-glass has a motion by hand, for producing distinct vision

of the wire; and distinct vision of the dots is procured by a similar adjustment of the whole microscope.

The first step towards sizing the roller, is to compute its diameter according to the measure of the circle, and to reduce it agreeably thereto, taking care to leave it a small matter too large. The second step is, after having brought the roller into its place in the Plate B B, to make a mark upon the surface of the circle near the edge, and a similar one upon the roller, exactly opposite each other; then carry the apparatus forward with a steady hand, until the roller has made sixteen revolutions: If, now, the mark upon the roller, by having over-reached the one upon the circle, shews it to be much too large, take it out of the frame and reduce it by turning accordingly: When by repeating this, it is found to be very near, it may be turned about .001 of an inch smaller on the lower edge, and so far its preparation is completed. The third and last step is, the use and adaptation of the two microscopes; one of these must take its position at H in Fig. 1, viewing a small well defined dot made for the purpose on the circle; the other, not represented in the figure, must also be fixed to the main plate of Fig. 1, as near to the former as possible, but viewing one of the divisions on the roller. With a due attention to each microscope, it will now be seen to the greatest exactness when, by raising or depressing the roller, its commensurate diameter is found.

Fig. 3 is a representation of the apparatus for transferring the divisions of the roller to the circle. It consists of two slender bars, which, being seen edgewise in the figure, have only the appearance of narrow lines; but, when looked at from above, they resemble the form of the letter A. They

are fastened to the main frame, as at W and Z, by short pillars, having also the off leg of the angle secured in the same manner; Y is a fine conical steel point for making the dots, and X is a feeler, whereby the point Y may be pressed down with a uniform force, which force may be adjusted, by bending the end of the bar just above the point, so as to make the dots of the proper size. The point Y yields most readily to a perpendicular action; but is amply secured against any eccentric or lateral deviation.

The apparatus, so far described, is complete for laying our foundation, *i. e.* making 256 primary dots; no matter whether with perfect truth, or not, as was said respecting the divisions of the roller; precision in either is not be expected, nor wished; but it is of some importance, that they should be all of the same size, concentric, small, and round. They should occupy a position very near the extreme border of the circle, as well to give them the greatest radius possible, as that there should be room for the stationary microscope and other mechanism, which will be described hereafter.

It must be noticed, that there is a clamp and adjusting screw attached to the main plate of Fig. 1; but, as it differs in no respect from the usual contrivances for quick and slow motion, it has been judged unnecessary to incumber the drawing with it.

Now, the roller having been adjusted, with one microscope H upon its proper dot on the circle, and the other microscope at the first division on the roller; place the apparatus of Fig. 3 so that the dotting point Y may stand directly over the place which is designed for the beginning of the divisions. In this position of things, let the feeler X be pressed down, until its

lower end comes in contact with the circle; this will carry down the point, and make the first impression, or primary dot, upon the circle; unclamp the apparatus, and carry it forwards by hand, until another division of the roller comes near the wire of the microscope; then clamp it, and with the screw motion make the coincidence complete; where again press upon the feeler for the second dot: proceed in this manner until the whole round is completed.

From these 256 erroneous divisions, by a certain course of examination, and by computation, to ascertain their absolute and individual errors, and to form these errors into convenient tables, is the next part of the process, and makes a very important branch of my method of dividing.

The apparatus must now be taken off, and the circle mounted in the same manner, that it will be in the Observatory. The two microscopes, which have divided heads, must also be firmly fixed to the support of the instrument, on opposite sides, and their wires brought to bisect the first dot, and the one which should be 180° distant. Now, the microscopes remaining fixed, turn the circle half round, or until the first microscope coincides with the opposite dot; and, if the other microscope be exactly at the other dot, it is obvious that these dots are 180° apart, or in the true diameter of the circle; and if they disagree, it is obvious that half the quantity by which they disagree, as measured by the divisions of the micrometer head, is the error of the opposite division; for the quantity measured is that by which the greater portion of the circle exceeds the less. It is convenient to note these errors + or —, as the dots are found too forward or too backward, according to the numbering of the degrees; and for the purpose

of distinguishing the + and — errors, the heads, as mentioned before, are numbered backwards and forwards to fifty. One of the microscopes remaining as before, remove the other to a position at right angles; and, considering for the present both the former dots to be true, examine the others by them; *i. e.* as before, try by the micrometer how many divisions of the head the greater half of the semi-circle exceeds the less, and note half the quantity + or —, as before, and do the same for the other semi-circle. One of the micrometers must now be set at an angle of 45° with the other, and the half differences of the two parts of each of the four quadrants registered with their respective signs. When the circle is a vertical one, as in the present instance, it is much the best to proceed so far in the examination with it in that position, for fear of any general bending or spring of the figure; but, for the examination of smaller arcs than 45°, it will be perfectly safe, and more convenient, to have it horizontal; because the dividing apparatus will then carry the micrometers, several perforations being made in the plate B for the limb to be seen through at proper intervals. The micrometers must now be placed at a distance of 22° 30′, and the half differences of the parts of all the arcs of 45° measured and noted as before; thus descending by bisections to 11° 15′, 5° 37′ 30″, and 2° 48′ 45″. Half this last quantity is too small to allow the micrometers to be brought near enough; but it will have the desired effect, if they are placed at that quantity and its half, *i. e.* 4° 13′ 7″,5; in which case the examination, instead of being made at the next, will take place at the next division but one, to that which is the subject of trial. During the whole of the time that the examination is made, all the dots, except the

one under examination, are for the present supposed to be in their true places; and the only thing in this most important part of the business, from first to last, is to ascertain with the utmost care, in divisions of the micrometer head, how much one of the parts of the interval under examination exceeds the other, and carefully to tabulate the half of their difference.

I will suppose that every one, who attempts to divide a large astronomical instrument, will have it engraved first. Dividing is a most delicate operation, and every coarser one should precede it. Besides, its being numbered is particularly useful to distinguish one dot from another; thus, in the two annexed tables of errors, the side columns give significant names to every dot, in terms of its value to the nearest tenth of a degree, and the mistaking of one for another is rendered nearly impossible.

The foregoing examination furnishes materials for the construction of the table of half differences, or apparent errors.* The first line of this table consists of two varieties; *i. e.* the micrometers were at 180° distance for obtaining the numbers which fill the columns of the first and third quadrant; and at 90°, for those of the second and fourth quadrant. The third variety makes one line, and was obtained with a distance of 45°: the fourth consists of two lines, with a distance of 22° 30′: the fifth of four lines, with a distance of 11° 15′: the sixth of eight lines, with a distance of 5° 37′ 30″: the seventh of sixteen lines, with a distance of 2° 48′ 45″: and the eighth and

* If the table of real errors be computed as the work of examination proceeds, there will be no occasion for this table at all; but, I think it best not to let one part interfere with another, and therefore I examine the whole before I begin to compute.

last variety, being the remainder of the table, consists of thirty two lines, and was obtained with a distance of $4^\circ\ 13'\ 7'',5$.

The table of apparent errors, or half differences, just explained, furnishes data for computing the table of real errors. The rule is this; let a be the real error of the preceding dot, and b that of the following one, and c the apparent error, taken from the table of half differences, of the dot under investigation; then is $\frac{a+b}{2} + c =$ its real error. But, as this simple expression may not be so generally understood by workmen as I wish, it may be necessary to say the same thing less concisely. If the real errors of the preceding and following dots are both +, or both —, take half their sum and prefix thereto the common sign; but, if one of them is +, and the other —, take half their difference, prefixing the sign of the greater quantity: again, if the apparent error of the dot under investigation has the same sign of the quantity found above, give to their sum the common sign, for the real error; but if their signs are contrary, give to their difference the sign of the greater for the real error. I add a few examples.

Example 1.

For the first point of the second quadrant.

Real error of the first point of the first quadrant -	0,0
Real error of the first point of the third quadrant	— 6,9
Half sum or half difference - -	— 3,4
Apparent error of the dot under trial -	+ 12,2
Real error - - - -	+ 8,8

Example 2.

For the point 45° of the second quadrant.

Real error of the first point of the quadrant -	+ 8,8
Real error of the last point of the quadrant -	— 6,9
Half difference - - - - -	+ 0,9
Apparent error of the dot under trial -	— 8,9
Real error - - - - - -	— 8,0

Example 3.

Point 88°,6, or last point, of the third quadrant.

Real error of the point 84°,4 of the third quadrant	— 21,0
Real error of the point 2°,8 of the fourth quadrant	— 2,9
Half sum - - - - -	— 11,9
Apparent error of the dot under trial -	— 4,0
Real error - - - - - -	— 15,9

Example 4.

Point 88°,6, or last, of the fourth quadrant.

Real error of the point 84°,4 of the fourth quadrant	— 21,6
Real error of the point 2°,8 of the first quadrant -	— 10,2
Half sum - - - - -	— 15,9
Apparent error of the dot under trial -	+ 9,5
Real error - - - - - -	— 6,4

It is convenient, in the formation of the table of real errors, that they should be inserted in the order of the numbering of the degrees on their respective quadrants; although their computation necessarily took place in the order in which the examination was carried on, or according to the arrangement in the table of apparent errors. The first dot of the first

quadrant having been assumed to be in its true place, the first of the third quadrant will err by just half the difference found by the examination; therefore these errors are alike in both tables. The real error of the first dot of the second quadrant comes out in the first example; that of the fourth was found in like manner, and completes the first line. It is convenient to put the error of the division 90° of each quadrant at the bottom of each column, although it is the same as the point 0° on the following quadrant. The line of 45° is next filled up; the second example shews this; but there is no occasion to dwell longer upon this explanation; for every one, who is at all fit for such pursuits, will think what has already been said fully sufficient for his purpose. However, I will just mention that there can be no danger, in the formation of this table, of taking from a wrong line the real errors which are to be the criterion for finding that of the one under trial; because they are in the line next to it; the others, which intervene in the full table, not being yet inserted. The last course of all is, however, an exception; for, as the examining microscopes could not be brought near enough to bisect the angle 2° 48′ 45″, recourse was had to that quantity and its half; on which account the examination is prosecuted by using errors at two lines distance, as is shewn in the two last examples.

When the table of real errors is constructed, the other table, although it is of no further use, should not be thrown away; for, if any material mistake has been committed, it will be discovered as the operation of dividing is carried on; and, in that case, the table of apparent errors must be had recourse to; indeed, not a figure should be destroyed until the work is done.

Respecting the angular value of the numbers in these tables, it may be worth mentioning, that it is not of the least importance; 100 of them being comprised in one revolution of the micrometer screw; and, in the instance before me, 5,6 of them made no more than a second. It is not pretended that one of these parts was seen beyond a doubt, being scarcely $\frac{1}{50000}$ of an inch, much less the tenths, as exhibited in the tables; but, as they were visible upon the micrometer heads, it was judged best to take them into the account.

Having now completed the two first sections of my method of dividing; namely, the first, which consists of making 256 small round dots; and the second, in finding the errors of those dots, and forming them into a table; I come now to the third and last part, which consists in using the erroneous dots in comparison with the tabulated errors, so as ultimately to make from them the true divisions.

It will here be necessary to complete the description of the remaining part of the apparatus. And first, a little instrument which I denominate a subdividing sector presents itself to notice. From all that has hitherto been said, it must have been supposed that the roller itself will point out, upon the limb of the instrument to be divided, spaces corresponding to others previously divided upon itself, as was done in setting off the 256 points: but, to obviate the difficulty of dividing the roller with sufficient exactness, recourse was had to this sector; which also serves the equally important purpose of reducing the bisectional points to the usual division of the circle. This sector is represented in full dimensions by Fig. 5: it is formed of thin brass, and centred upon the axis at A, in contact with the upper surface of the roller: it is capable of

being moved round by hand; but, by its friction upon the axis and its pressure upon the roller, it is sufficiently prevented from being disturbed by accident. An internal frame B B, to which the arc C C is attached, moves freely in the outer one, and by a spring D is pushed outwards, while the screw E, whose point touches the frame B, confines the arc to its proper radius. The arc of this sector is of about four times greater radius than the roller, and upon it are divided the spaces which must be transferred to the instrument, as represented on a magnified scale by Fig. 4. Now, the angle of one of the spaces of the circle will be measured by sixteen times its angular value upon the sectorial arc, or $22^\circ\ 30'$; but this does not represent any number of equal parts upon the instrument, whose subdivisions are to be $5'$ each; for $\frac{1^\circ\ 24'\ 22''5}{5}$ is exactly $16\frac{7}{8}$, therefore so many divisions are exactly equal to a mean space between the dots whose errors have been tabulated. Let, therefore, the arc of the sector be divided into 16 spaces of $1^\circ\ 20'$ each, and let a similar space at each end be subdivided into eight parts of $10'$ each, as in Fig. 4; we shall then have a scale which furnishes the means for making the true divisions, and an immediate examination at every bisectional point.

I have always divided the sector from the engine, because that is the readiest method, and inferior to none in point of accuracy, where the radius is very short; but, as it is more liable than any other to centrical error, the adjustment of the arc by the screw E becomes necessary: by that adjustment, also, any undue run in the action of the roller may be reduced to an insensible quantity.*

* See note page 130.

When the utmost degree of accuracy is required, I give the preference to dividing by lines, because they are made with a less forcible effort than dots are; and also because, if any small defect in the contexture of the metal causes the cutter to deviate, it will, after passing the defective part, proceed again in its proper course, and a partial crookedness in the line will be the only consequence; whereas a dot, under similar circumstances, would be altogether displaced. But, on the other hand, where accuracy has been out of the question, and only neatness required, I have used dots; and I have done so, because I know that when a dot and the wire which is to bisect it are in due proportion to each other, (the wire covering about two thirds of the dot) the nicest comparison possible may be obtained. It may be further observed, that division by lines is complete in itself; whereas that by dots requires lines to distinguish their value.

On the upper side of Fig. 1 is represented the apparatus for cutting the divisions. It consists of three pieces JKL, jointed together so as to give to the cutter an easy motion for drawing lines directly radiating from the centre, but inflexible with respect to lateral pressure; *dd* are its handles. The cutting point is hidden below the microscope H; it is of a conical form, and were it used as a dotting point, it would make a puncture of an elliptical shape, whose longer diameter would point towards the centre. This beautiful contrivance, now well known, we owe to the ingenuity of the late Mr. HINDLEY of York; it was borrowed by Mr. RAMSDEN,* and applied with the best effect to his dividing engine.

* This I learned from that most accurate artist Mr. JOHN STANCLIFFE, who was himself apprentice to HINDLEY.

It might have been mentioned sooner, that in the instance which I have selected as an example of my dividing, the operation took place when the season of the year, and the smoke of London, had reduced the day to scarcely six hours of effective light; and rather than confine my labours within such narrow limits, I determined to shut out the day-light altogether. Fig. 7 shews the construction of the lanterns which I used. A very small wick gave sufficient light, when kept from diverging by a convex lens; while the inclining nossel was directed down exactly upon the part looked at, and the light, having also passed through a thin slice of ivory, was divested of all glare. I enter into this description, because, I think, I never saw my work better, nor entirely to so much advantage as in this instance; owing, perhaps, to the surrounding darkness allowing the pupil of the eye to keep itself more expanded, than when indirect rays are suffered to enter it. The heat from a pair of these lanterns was very inconsiderable, and chiefly conducted along with the smoke up the reclining chimney.

Previous to cutting the divisions, the parts now described must be adjusted. The cutting apparatus must be placed with the dividing point exactly at the place where the first line is intended to be drawn, and clamped, so that the adjusting screw may be able to run it through a whole interval. The microscope H must be firmly fixed by its two pillars *b b* to the main frame, with its micrometer head at *zero;* and with its only wire in the line of the radius, bisecting the first of the 256 dots. And it should be observed, that the cutting frame and this must not vary respecting each other, during the time that the divisions are cut; for any motion that took place in either

would go undiminished to the account of error. The microscope I is also fastened to the main frame; but it is only required to keep its position unvaried, while the divisions of the sector pass once under its notice; for it must have its wires adjusted afresh to these divisions at every distinct course. The microscope I has two wires, crossing each other at an angle of about 40°; and these are to be placed so as to make equal angles with the divisions of the sector, which are not dots, but lines. The sectorial arc must also be adjusted to its proper radius by the screw E, Fig. 5; *i. e.* while the main frame has been carried along the circle through a mean interval shewn by H, the sector must have moved through exactly $16\frac{7}{8}$ of its divisions, as indicated by I.*

Things being in this position; after having given the parts time to settle, and having also sufficiently proved the permanence of the micrometer H and the cutting frame with respect to each other, the first division may be made; then, by means of the screw for slow motion, carry the apparatus forward, until the next line upon the sector comes to the cross wires of I; you then cut another division, and thus proceed until the 16th division is cut, = 1° 20′: Now the apparatus wants to be carried

* For the sake of simplicity, the account of the process is carried on as if the roller measured the mean interval without error: But it was said (Page 107) that the roller, in a continued motion quite round the circle, would in some part of its course err by 30″ or more; therefore, when that is the case, an extreme run of the roller cannot agree with a mean interval of the circle nearer than $\frac{30''}{128} = 0'',23$; and most probably this kind of error will on some intervals amount to double that quantity. It therefore becomes matter of prudent precaution to examine every interval previous to making the divisions; and, where necessary, to adjust the sector, so that its arc may exactly measure the corresponding interval as corrected by the tabulated errors.

further, to the amount of $\frac{7}{8}$ of a division, before an interval is complete; but at this last point no division is to be made; we are here only to compare the division on the sector with the corresponding dot upon the instrument: This interval, however, upon the circle will not be exactly measured by the corresponding line of the sector, which has been adjusted to the mean interval, for the situation of the dot 1°,4 is too far back, as appears by the table of real errors, by — 4,8 divisions of the micrometer head. The range of the screw for slow motion must now be restored, the cross wires of H set back to — 4,8 divisions, and the sector moved back by hand, but not to the division 0 where it began before; for, as it left off in the first interval at $\frac{7}{8}$ of a division, it has to go forwards $\frac{1}{8}$ more before it will arrive at the spot where the 17th division of the instrument 1° 25′ is to be made, so that in this second course it must begin at $\frac{1}{8}$ short of 0: Go through this interval as before, making a division upon the circle at every one of the 16 great divisions of the sector; and H should now reach the third dot, allowing for a tabular error of — 10,2 when the division $\frac{6}{8}$ths of the sector reaches the cross wires of I. It would be tedious to lead the reader through all the variety of the sector, which consists of eight courses; and it may be sufficient to observe, that at the commencement of every course, it must be put back to the same fraction of a division which terminated its former one; and that the wire of the micrometer H must always be set to the tabular error belonging to every dot, when we end one interval and begin another. The eight courses of the sector will have carried us through $\frac{1}{32}$ part of the circle, 11° 15′, and during this time, the roller will have proceeded through half a revolution; for

its close contact with the limb of the circle does not allow it to return with the sector when the latter is set back at every course. Having in this manner proceeded, from one interval to another, through the whole circle, the micrometer at last will be found with its wire, at *zero*, on the dot from which it set out; and the sector, with its 16th division, coinciding with the wires of its microscope.

Having now given a faithful detail of every part of the process of dividing this circle, I wish to remind the reader that, by verification and correction at every interval, any erroneous action of the roller is prevented from extending its influence to any distant interval. It will be further observed, that the subdividing sector magnifies the work; that by means of its adjustable arc, it makes the run of the roller measure its corresponding intervals upon the circle; and, without foreign aid, furnishes the means of reducing the bisectional intervals to the usual division of the circle. Furthermore, the motion of the wire of the micrometer H, according to the divisions of its head and corresponding table of errors, furnishes the means of prosecuting the work with nearly the same certainty of success, as could have happened, had the 256 points been (which in practice is quite impossible) in their true places.

Now, the whole of my method of dividing being performed by taking short measures with instruments which cannot themselves err in any sensible degree, and, inasmuch as those measures are taken, not by the hand, but by vision, and the whole performed by only looking at the work, the eye must be charged with all the errors that are committed until we come to cut the divisions; and, as in this last operation the

hand has no more to do than to guide an apparatus so perfect in itself, that it cannot be easily made to deviate from its proper course, I would wish to distinguish it from the other methods by denominating it, DIVIDING BY THE EYE.*

The number of persons at all capable of dividing originally have hitherto been very few; the practice of it being so limited, that, in less than twice seven years, a man could hardly hope to become a workman in this most difficult art. How far I shall be considered as having surmounted these difficulties, I know not; but if, by the method here revealed,

* I must here remark, that SMEATON has represented the greatest degree of accuracy that can be derived from vision, in judging of the coincidence of two lines at $\frac{1}{4000}$ part of an inch. From this it may fairly be inferred, that he had not cultivated the power of the sight, as he had done that of the touch; the latter of which, with that ability which appeared in all his works, he rendered sensible to the $\frac{1}{60000}$ part of an inch. Were materials infinitely hard, no bounds could be set to the precision of contact; but taking things as they are, the different degrees of hardness in matter, may be considered as a kind of magnifying power to the touch, which may not unaptly be compared with the assistance which the eye receives from glasses. It is now quite common to divide the seaman's sextant to 10″, and a good eye will estimate the half of it; which, on an eight inch radius is scarcely $\frac{1}{10000}$ of an inch. This quantity, small as it is, is rendered visible by a glass of one inch focal length; and such is the certainty with which these quantities are seen, that a seaman will sometimes complain that two pair of these lines will coincide at the same time; and that may happen, and yet no division of his instrument err, by more than $\frac{1}{20000}$ part of an inch. All this is applicable to judging of the coincidence of *lines* with each other, and furnishes not the most favourable display of the accuracy of vision. But with the microscopes here described, where the wire bisects the image of a dot, or a cross wire is made to intersect the image of a line, by an eye practised in such matters, a coincidence may undoubtedly be ascertained to $\frac{1}{50000}$ part of an inch. I am of opinion that as small a quantity may be rendered visible to the eye, as can by contact be made sensible to the touch; but whether Mr. SMEATON's $\frac{1}{60000}$ and my $\frac{1}{50000}$ be not the same thing, I will not determine; the difference between them, however, is what he would no more have pretended to feel, than I dare pretend to see.

I have not rendered original dividing almost equally easy with what copying was before, I have spent much labour, time, and thought in vain. I have no doubt indeed, that any careful workman who can divide in common, and has the ability to construct an astronomical instrument, will, by following the steps here marked out, be able to divide it, the first time he tries, better than the most experienced workman, by any former method.

If, instead of subdividing with the roller, the same thing be performed with the screw, it will not give to dividing by the eye any very distinctive character: I have practised this on arcs of circles with success, the edge being slightly racked, the screw carrying forward an index with the requisite apparatus, and having a divided micrometer head; the latter answers to the subdividing sector, and, being used with a corresponding table of errors, forms the means of correcting the primitive points; but the roller furnishes a more delicate action, and is by far more satisfactory and expeditious.

It is known to many that the six feet circle, which I am now at work upon for our Royal Observatory, is to be divided upon a broad edge, or upon a surface at right angles to the usual plane of division: The only alterations, which will on that account be required, are, that the roller must act upon that plane which is usually divided upon; which roller, being elevated or depressed, may be adjusted to the commensurate radius without being made conical, as was necessary in the other case. The apparatus, similar to the other, must here be fixed immoveable to the frame which supports the circle; its position must be at the vertex, where also I must have my station; and the instrument itself must be turned around its axis, in its

proper vertical position, as the work proceeds. The above may suffice, for the present, to gratify those who feel themselves interested upon a subject which will be better understood, if I should hereafter have the honour of laying before the Royal Society a particular description of the instrument here alluded to; a task which I mean to undertake, when, after being fixed in the place designed for it, which I hope will be effected at no very distant period, it shall be found completely to answer the purposes intended.

Should it be required to divide a circle according to the centesimal division of the quadrant, as now recommended and used in France, we shall have no difficulty. The 100° of the quadrant may be conveniently subdivided into 10 each, making 4000 divisions in the whole round. The 256 bisectional intervals, the two tables of errors, and the manner of proceeding and acting upon them will be exactly the same as before, until we come to cut the divisions; and for this purpose we must have another line divided upon the sector. For $\frac{1}{4000}$ part of the circle being equal to $5',4$ of the usual angular measure $\frac{1^\circ\, 24'\, 22'',5}{5',4} = 15\frac{5}{8}$ divisions; and just so many will be equivalent to one of the intervals of the circle. The value of one of the great divisions of the sector will be $1^\circ\ 26'\ 24''$, and that of the $\frac{1}{8}$ parts, which are to be annexed to the right and left as before, will be $10'\ 48''$, therefore divisible by the engine. Should any astronomer choose to have both graduations upon his instrument, the additional cost would be a mere trifle, provided both were done at the same time.

It must already have been anticipated, that dividing by the eye is equally applicable to straight lines as it is to circles.

An apparatus for this purpose should consist of a bar of brass, three quarters of an inch thick, and not less than three inches broad; six feet may do very well for the length; it may be laid upon a deal plank strengthened by another plank screwed edgewise on its lower surface. The bar should be planed, on both its edges and on its surface, with the greatest exactness; and it will be better, if it has a narrow slip of silver, inlaid through its whole length, for receiving the dots. An apparatus nearly similar to the other should slide along its surface, carrying a roller, whose circumference is 12,8 inches, and turned a little conical for the sake of adjustment. The roller may be divided into 32 parts, each of which when transferred to the bar will give intervals of 0,4 of an inch each: The angle of the subdividing sector should of course be 11° 15′, and subdivided into four parts, which will divide the inch into tenths: The surface may also receive other lines, with subdivisions suited to the different purposes for which it may be wanted. The revolutions of the roller and its $\frac{1}{32}$ parts must be dotted upon the bar; taking care, by sizing the roller, to come as near the true standard measure as possible: When this is done, compare the extent of the greater bisectional number that is contained in the length; *i.e.* 128 intervals or 51,2 inches, with the standard measure; noting the difference as indicated by the micrometer heads: The examination and construction of the table of errors may then be conducted just as was done for the circle.

Being now ready for the performance of its work, the scale to be divided must be laid alongside of the bar, and the true divisions must be cut upon it by an appeal, as before, to the erroneous dots on the bar, corrected by a corresponding table

of errors. The apparatus, remaining entire in the possession of the workman, with its primitive dots, the table of errors, &c. is ready for dividing another standard, which will be precisely similar to others that have been, or may be, divided from it. It may be considered, indeed, as a kind of engine; and, as it is not vitiated by the coarse operation of racking with a screw, but performed by only looking at the work, the method will command about three times the accuracy that can be derived from the usual straight-line dividing engine. Should it be asked, if an engine thus appointed would succeed for dividing circles? I answer, Yes; but I would not recommend it; because, beyond a certain extent of radius, it is not necessary; for the errors, which would be introduced into the work by the violence of racking a large wheel, are sufficiently reduced by the comparative shortness of the radius of such instruments as we divide by that method: And, what is still more to the purpose, the dividing engine is four times more expeditious, and bears rough usage better. I cannot quit the subject of dividing straight lines without observing, that I never had my apparatus complete. The standard which I made for Sir GEORGE SHUCKBURG EVELYN in 1796 was done by a mere make-shift contrivance, upon the principle of dividing by the eye; how I succeeded may be seen in Sir GEORGE'S papers on Weights and Measures (Phil. Trans. for 1798). I made a second, some years after, for Professor PICTET of Geneva, which became the subject of comparison with the new measure of France, before the National Institute; and their report, drawn up by Mr. PICTET, has been ably re-stated and corrected by Dr. YOUNG, as published in the Journals of the Royal Institution. I made a third for the Magistrates of

Aberdeen. I notice the two latter, principally to give myself an opportunity of saying that, if those three scales were to be compared together, notwithstanding they were divided at distant periods of time, and at different seasons of the year, they would be found to agree with each other, as nearly as the different parts of the same scale agree.

I hope I may here be allowed to allude to an inadvertency which has been committed in the paper mentioned above; and which Sir GEORGE intended to have corrected, had he lived to conclude his useful endeavours to harmonize the discordant weights and measures of this country. The instruments which he has brought into comparison are, his own five feet standard measure and equatorial; General ROY's forty-two inch scale; the standard of Mr. AUBERT; and that of the Royal Society. The inadvertency is this: In his equatorial, and the standard of the Royal Society, he has charged the error of the most erroneous extent, when compared with the mean extent, alike to both divisions; *i. e.* he has supposed one of the divisions, which bound the erroneous extent, to be too much to the right, and the other too much to the left, and that by equal quantities: This is certainly a good natured way of stating the errors of work; and perhaps not unjustly so, where the worst part has been selected; but, in the other three instances, namely, in General ROY's, Mr. AUBERT's, and his own standard, he has charged the whole error of the most erroneous extent to one of the bounding lines.

I was well confirmed in my high opinion of the general accuracy of BIRD's dividing, when, last winter,* I measured the chords of many arcs of the Greenwich quadrant: That instrument has indeed suffered both from a change in its figure,

* This paper was written in June 1808.

and from the wearing of its centre; but the graduation, considering the time when it was done, I found to be very good. Sir GEORGE in his Paper upon the Equatorial (Phil. Trans. for 1793), after some compliments paid to the divider of his instrument, says, " the late Mr. JOHN BIRD seems to have ad-" mitted a probable discrepancy in the divisions of his eight " feet quadrant amounting to 3″;" and he refers to BIRD on the construction of the Greenwich quadrant. This quantity being three times as great as any errors that I met with, I was lately induced to inquire how the matter stood. BIRD, in the paper referred to, says, " in dividing this instrument I " never met with an inequality that exceeded one second. I " will suppose that in the 90 arch this error lay towards the " left hand, and in the 96 arch that it lay towards the right, " it will cause a difference between the two arches of two " seconds; and, if an error of one second be allowed to the " observer in reading off his observation, the whole amount " is no more than three seconds, which is agreeable to what " I have heard, &c." Sir GEORGE's examination of his own Equatorial furnishes me with the means of a direct comparison: In his account of the declination circle, we find an error + 2″,35, and another — 1″,5; to these add an error of half a second in each, for reading off, which Sir GEORGE also admits, we shall then have a discrepancy of 4″,85; but, as the errors of reading off are not errors of division, let them be discharged from both, and the errors will then stand, for the quadrant 2″, and for the circle 3″,85. As the radius of the former, however, is four times greater than that of the latter, it will appear, by this mode of trial, that the Equatorial is rather more than twice as accurately divided as the quadrant,

In doing justice to BIRD in this instance, I have only done as I would be done by; for, should any future writer set me back a century on the chronological scale of progressive improvement, I hope some one will be found to restore me to my proper niche. I now subjoin a re-statement of the greatest error of each of the instruments that are brought into comparison by Sir GEORGE, after having reduced them all by one rule; viz. allowing each of the two points which bound the most erroneous extent to divide the apparent error equally between them. They are expressed in parts of an inch, and follow each other in the order of their accuracy.

Sir GEORGE SHUCKBURG's 5 feet standard	,000165
General ROY's scale of 42 inches - -	,000240
Sir GEORGE's Equatorial, 2 feet radius -	,000273
The Greenwich quadrant, 8 feet radius -	,000465
Mr. AUBERT's standard, 5 feet long -	,000700
* The Royal Society's standard 92 inches long	,000795

For the justness of the above statement I consider my name as pledged; requesting the permission to say, that if on the result of each respective examination, as here presented, there could have been more than one opinion, it would not have appeared here. I am further prompted to add, that the above comparative view presents one circumstance to our notice, which cannot do less than gratify every individual who is at all conversant in these matters; I mean, the high rank which General ROY's scale takes in the list; that scale having been made the agent in measuring the base line of our national trigonometrical survey.

* This is the same which Mr. BIRD used in dividing his eight feet mural quadrants, and was presented to the Royal Society by BIRD's executors.

To return, finally, to the dividing of circles; I must state, as matter of precaution, that great care should be taken during the turning of the outer edge, to have the circle of the same temperature; for one part may be expanded by heat, or contracted by cold, so much more than another, as to cause the numbers in the tables of errors to be inconveniently large. A night is not more than sufficient for allowing the whole to take the same temperature, after having been handled by the workmen; and the finishing touch should be given within a short space of time. But, if the effects of temperature are to be regarded in turning a circle, it is of tenfold more importance to attend to this circumstance, while the examination of the larger arcs of the instrument is carried on; for it is absolutely necessary that, during this time, the whole circle should be of the same heat exactly. Few workmen are sufficiently aware of this: They generally suppose the expansion of metals to be a trifle which need not be regarded in practice; and wonder how the parts of a circle can be differently heated without taking pains to make it so. One degree of FAHRENHEIT's thermometer indicates so small a portion of heat that, in such places as workmen are usually obliged to do their business in, it is not very easy to have three thermometers attached to different parts of a large instrument, shewing an equality of temperature within that quantity: Yet so necessary is correctness in this respect, that if a circle has the vertex one degree warmer than its opposite, and if this difference of temperature be regularly distributed from top to bottom, the upper semi-circle will actually exceed the lower by 2″: And, if such should happen to be the case while the examination of the first dot of the third quadrant is made,

the regularity of the whole operation would thereby be destroyed.

It may not be improper to remark, that dividing by the eye does not require a more expensive apparatus than the operation of dividing by hand; and, indeed, less so when the scale of inches is deemed necessary. The method by adjustment is still more expensive, requiring whatever tools BIRD's method requires, and, in addition to these, a frame and microscopes, somewhat similar to those for dividing by the eye.

It is somewhat more difficult to give a comparative estimate of the time which the different methods of dividing require. I know that thirteen days of eight hours each, are well employed in dividing such a circle by my method; about fifty-two days would be consumed in doing the same thing by BIRD's method; and I think I cannot err much when I state the method by adjustment, supposing every dot to be tried, and that two-thirds of them want adjusting, to require about one hundred and fifty of such days.

The economy of time (setting aside the decided means of accuracy) which the above estimate of its application offers to view, will, I think, be considered of no little moment. By the rising artist who may aspire at excellence, it will at least, and I should hope, with gratitude, be felt in the abbreviation of his labours. To me, indeed, the means of effecting this became indispensible; and it has not been without a sufficient sense of its necessity, that I have been urged to the progressive improvement and completion of these means, as now described. It is but little that a man can perform with his own hands alone; nor is it on all occasions, even in frames of firmer texture than my own, that he can decisively command their

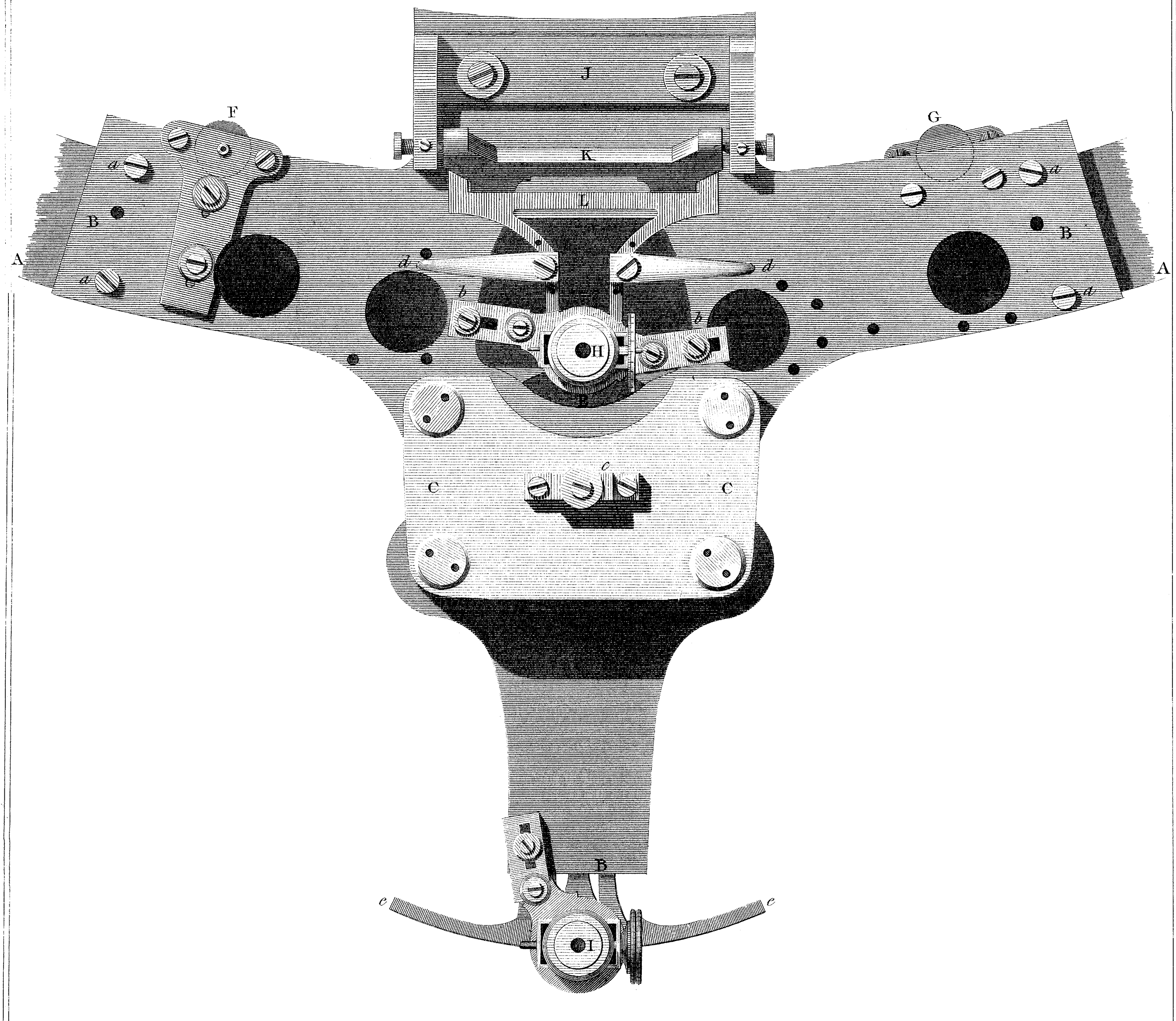
adequate, unerring, use. And I must confess that I never could reconcile it to what I hold as due to myself, as well as to a solicitous regard for the most accurate cultivation of the science of astronomy, to commit to others an operation requiring such various and delicate attentions, as the division of my instruments.

That my attentions on this head have not failed to procure for me the notice and patronage of men whose approbation makes, with me, no inconsiderable part of my reward, I have to reflect on with gratitude and pleasure: And as I look with confidence to the continuance of that patronage so long as the powers of execution shall give me the inclination to solicit it, I cannot entertain a motive which might go to extinguish the more liberal wish of pointing out to future ingenuity a shorter road to eminence; sufficiently gratified by the idea of having in the present communication, contributed to facilitate the operations, and to aid the progress of art (as far as the limited powers of vision will admit) towards the point of perfection.

Table of apparent Errors.

Name of the Dot.	First Quadrant.	Second Quadrant.	Third Quadrant.	Fourth Quadrant.	First Quadrant.	Second Quadrant.	Third Quadrant.	Fourth Quadrant.	Name of the Dot.
° 0,0	0	+ 12,2	− 6,9	+ 17,9	+ 4,6	+ 17,1	− 4,4	+ 17,3	° 1,4
45,0	− 21,3	− 8,9	16,7	− 29,6	− 5,2	− 9,7	8,9	− 6,4	4,2
22,5	1,6	2,2	1,0	2,7	0,0	3,8	1,0	4,7	7,0
67,5	+ 1,0	+ 15,6	0,0	+ 13,7	+ 1,0	+ 3,5	5,1	5,5	9,8
11,2	− 16,6	− 20,2	22,6	− 30,3	− 5,5	− 1,6	0,0	+ 1,2	12,7
33,7	4,0	4,2	13,2	23,1	7,6	7,6	4,2	− 2,3	15,5
56,2	16,9	22,2	17,0	22,7	9,4	3,9	0,0	5,3	18,3
78,7	30,8	16,6	31,3	30,3	+ 1,1	+ 12,1	+ 4,2	+ 4,3	21,1
5,6	2,7	8,6	4,1	10,1	12,3	0,9	6,2	14,4	23,9
16,9	11,5	11,3	11,2	16,1	− 5,7	− 6,2	1,1	− 11,2	26,7
28,1	9,0	7,4	5,8	14,3	+ 1,5	3,5	− 6,3	4,2	29,5
39,4	9,3	8,2	5,8	13,1	0,0	7,0	7,7	+ 1,4	32,3
50,6	4,2	6,6	8,2	4,4	1,5	+ 9,0	+ 3,0	4,3	35,2
61,9	4,3	8,4	12,5	4,4	− 8,6	− 5,9	− 2,0	− 6,7	38,0
73,1	7,6	10,0	13,6	9,7	3,3	+ 2,7	4,9	1,5	40,8
84,4	18,0	+ 6,0	16,3	7,1	+ 4,0	3,1	3,5	+ 1,0	43,6
2,8	3,4	− 7,5	8,9	2,1	13,5	10,5	+ 16,0	14,9	46,4
8,4	0,0	5,0	4,6	5,7	2,1	0,0	1,7	− 3,5	49,2
14,1	6,6	8,2	5,6	4,8	− 5,0	− 10,7	− 2,9	1,5	52,0
19,7	1,6	2,4	+ 1,0	2,5	4,2	7,9	2,2	7,2	54,8
25,3	3,7	8,2	− 2,9	2,5	4,0	3,0	2,5	1,0	57,7
30,9	+ 2,4	7,1	7,0	0,0	7,3	+ 6,2	6,1	1,5	60,5
36,6	− 5,9	+ 1,0	2,5	1,5	3,2	− 10,1	5,6	12,7	63,6
42,2	+ 3,1	1,9	5,8	+ 2,5	1,4	7,2	3,9	+ 2,2	66,1
47,8	7,1	5,2	+ 2,4	4,8	+ 11,2	+ 14,9	+ 21,2	7,2	68,9
53,4	− 5,6	− 6,0	− 5,0	− 6,1	− 7,1	− 1,0	− 8,9	− 11,7	71,1
59,1	10,7	+ 1,0	3,0	+ 1,4	5,3	1,2	6,6	2,7	74,5
64,7	7,9	− 18,0	10,7	− 9,0	7,2	9,9	+ 1,0	5,9	77,3
70,3	2,7	7,4	1,5	9,0	6,5	1,8	5,3	2,6	80,2
75,9	1,2	5,2	2,2	4,7	+ 4,4	+ 1,4	− 2,2	4,3	83,0
81,6	1,6	+ 1,7	0,0	2,0	− 20,8	− 0,0	11,4	+ 1,0	85,8
87,2	13,7	6,0	3,5	+ 5,6	+ 2,1	+ 11,0	4,0	9,5	88,6

Fig. 1.



Basire sc.

Fig. 2.

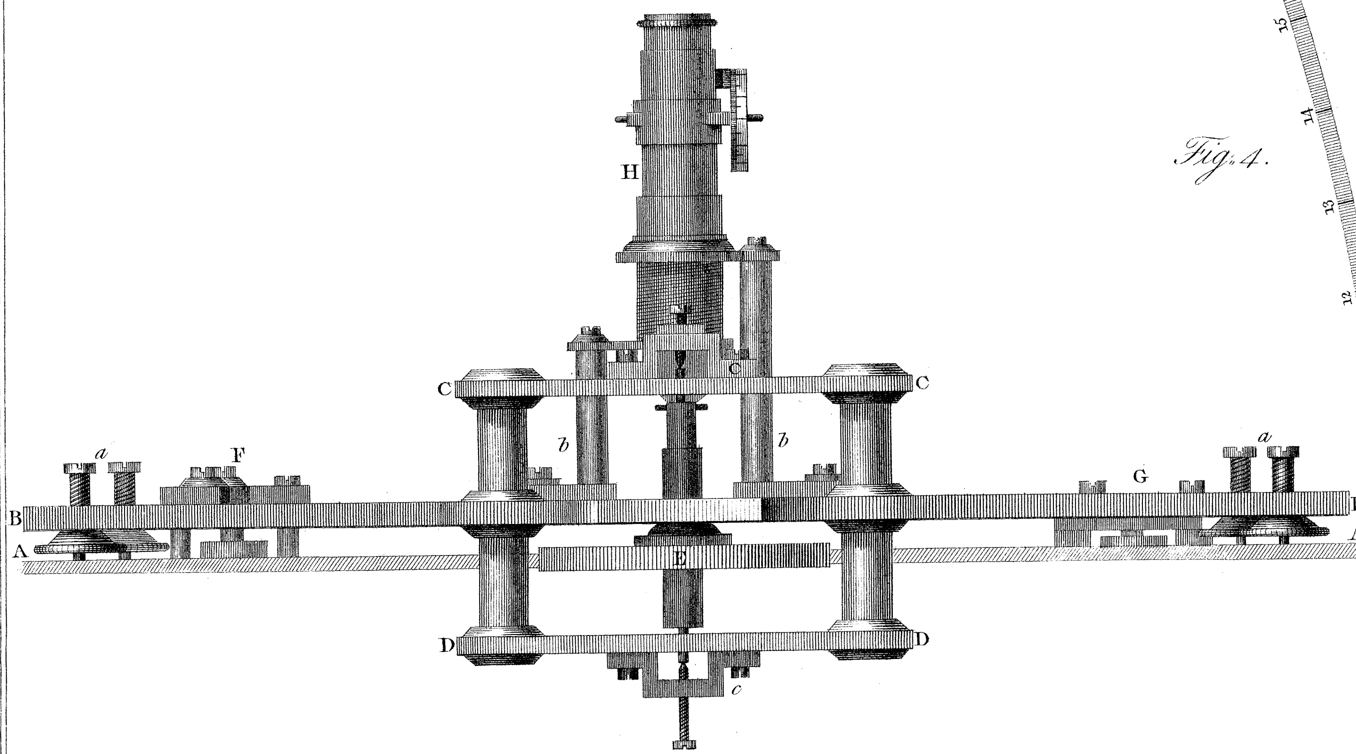


Fig. 4.

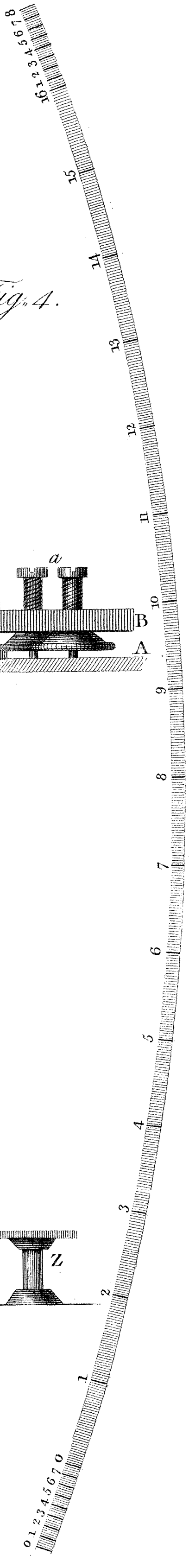
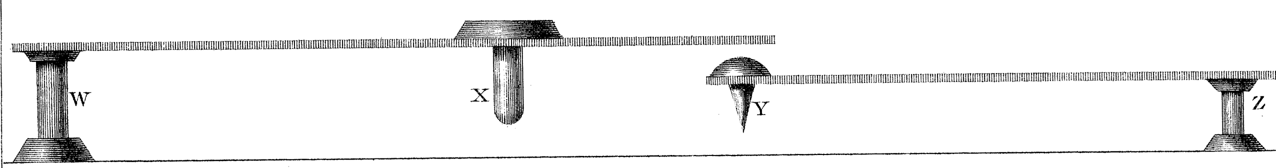


Fig. 3.



Basire sc.

Fig. 5.

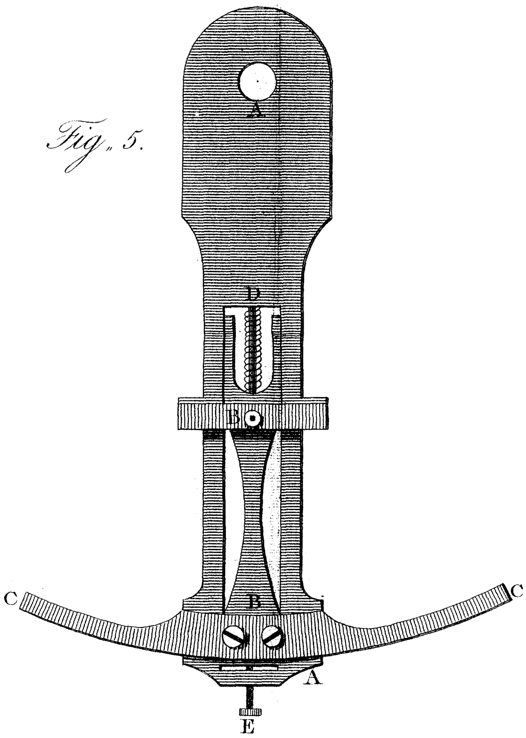


Fig. 6.

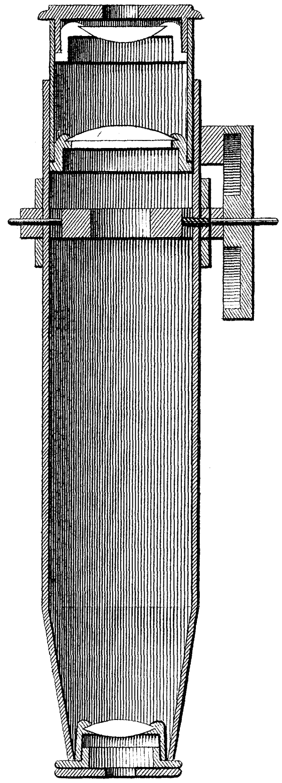
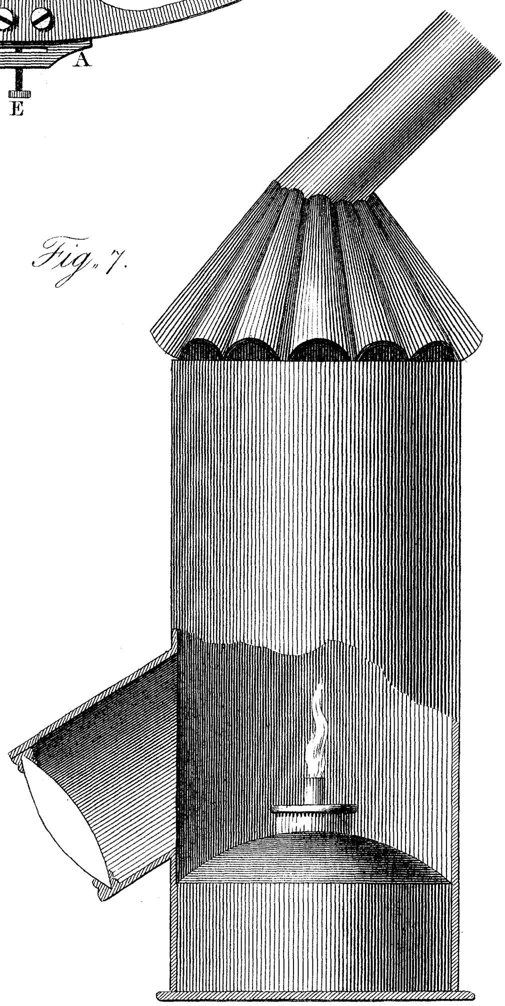


Fig. 7.



Basire sc.

Table of real Errors.

Name of the Dot	First Quadrant.	Second Quadrant.	Third Quadrant.	Fourth Quadrant.	First Quadrant.	Second Quadrant.	Third Quadrant.	Fourth Quadrant.	Name of the Dot.
° 0,0	0,0	+ 8,8	− 6,9	+ 14,4	− 16,9	− 8,0	− 13,4	− 22,4	° 45,0
1,4	− 4,8	− 0,6	16,0	5,9	8,7	5,5	9,7	16,1	46,4
2,8	10,2	9,3	24,0	− 2,9	14,3	9,6	17,4	22,3	47,8
4,2	13,8	15,1	28,3	12,8	22,3	17,9	19,9	33,8	49,2
5,6	13,7	12,5	23,3	16,1	26,0	21,6	26,7	31,9	50,6
7,0	15,9	16,8	28,7	19,4	25,5	26,0	23,6	28,9	52,0
8,4	17,6	19,6	32,0	27,0	32,0	27,8	30,3	38,3	53,4
9,8	21,4	16,1	35,5	30,7	34,0	27,3	29,1	35,2	54,8
11,2	21,6	16,7	31,5	26,5	26,8	22,1	24,0	32,6	56,2
12,7	27,9	21,6	32,2	28,6	29,6	24,5	29,7	29,8	57,7
14,1	31,1	26,8	37,5	34,4	33,7	17,7	27,2	24,6	59,1
15,5	28,5	22,7	30,2	26,8	30,2	15,6	29,3	26,5	60,5
16,9	27,3	20,5	32,4	32,7	19,2	15,3	24,1	19,4	61,9
18,3	29,9	18,2	24,2	25,7	21,5	14,6	18,8	23,7	63,3
19,7	20,2	13,5	20,6	22,2	19,0	21,5	22,4	17,4	64,7
21,1	22,4	5,9	22,1	24,0	18,8	19,9	22,8	17,1	66,1
22,5	10,0	1,8	10,9	6,7	3,0	+ 8,2	+ 0,7	+ 2,5	67,5
23,9	8,8	12,2	16,0	14,9	9,8	− 2,8	− 2,5	− 13,0	68,9
25,3	19,8	15,5	20,2	24,0	15,7	10,2	13,7	19,2	70,3
26,7	21,7	16,1	20,0	33,0	21,9	7,0	21,8	25,8	71,7
28,1	22,1	12,8	23,8	36,4	23,0	13,9	25,1	23,0	73,1
29,5	17,1	15,8	28,9	35,0	27,1	14,3	25,3	26,8	74,5
30,9	22,1	18,0	31,4	37,0	26,6	20,1	26,6	30,7	75,9
32,3	24,7	19,3	33,3	37,7	33,3	21,1	22,7	31,1	77,3
33,7	17,4	9,1	25,1	37,6	27,9	16,0	23,8	29,1	78,7
35,2	22,7	8,0	25,1	35,7	35,5	14,5	18,5	28,7	80,2
36,6	27,3	11,9	27,4	41,8	29,3	9,0	22,4	27,3	81,6
38,0	26,5	15,6	26,9	40,6	21,0	6,6	17,5	21,4	83,0
39,4	26,4	16,7	24,8	43,1	27,5	5,4	21,0	21,6	84,4
40,8	25,4	7,2	25,1	33,6	31,0	7,9	15,4	12,6	85,8
42,2	18,5	10,4	24,7	30,2	23,0	0,1	6,8	5,2	87,1
43,6	16,3	10,0	24,6	31,7	16,3	3,7	15,9	6,4	88,6
45,0	16,9	8,0	13,0	22,4	+ 8,8	6,9	+ 14,4	0,0	90,0